Summary. The simulation of a zero-offset section from multi-coverage seismic reflection data for 2-D media is a widely used seismic reflection imaging method that reduces the amount of data and enhances the signal-to-noise ratio. The aim of the common-reflection-surface stack method is not only to improve the resulting stack section but also to determine parameters that are useful with respect to a subsequent inversion. These additional parameters are attributes of hypothetical wavefronts observed at the surface.
The main advantage of the common-reflection-surface stack is the use of analytical formulae that describe the kinematic reflection moveout response for inhomogeneous media with curved interfaces but do not depend on a macro velocity model. An analytic reflection response fitting best an actual reflection event in the data set is determined by coherency analysis.
Although we applied the common-reflection-surface stack to various synthetic and real data sets, we restrict ourselves to a synthetic example. For a given model data-derived as well as model-derived (forward calculated) wavefront attributes are available. This enables us to verify the wavefront attributes determined by the common-reflection-surface stack method exposing a wide agreement with the expected results.

Introduction. Many conventional imaging methods require a sufficiently accurate macro velocity model to yield correct results. To calculate the respective operators (e. g. isochrons for dip moveout (DMO) correction or traveltime surfaces associated with hypothetical diffractors for pre-stack migration (PSM)) it is in addition necessary to perform ray tracing to obtain the traveltimes.
Our aim is to determine appropriate 2-D stacking operators without the knowledge of a macro velocity model and, consequently, without ray tracing. The “best” stacking operator is determined by means of coherency analysis (Taner and Koehler (1969)): we test a set of different stacking operators for the highest coherence obtained along the respective operator in the input data set.
For homogeneous models the stacking operator is the kinematic impulse response of a circular reflector segment in the subsurface, the *common reflection surface* (CRS). This reflector segment can be described by means of three parameters: its location, orientation, and curvature. Performing two hypothetical experiments with sources on the reflector segment yields wavefronts associated with two so-called *eigenwaves*. These wavefronts would be observed at the surface with well-defined attributes, namely the angles of emergence (which coincide for both eigenwaves) and the respective curvatures of the emerging hypothetical wavefronts. In other words, the common angle of emergence and the two curvatures uniquely define the considered three-parametric circular reflector segment.
Moving to the more general case of inhomogeneous models, these wavefront attributes can still be used to define the stacking operator assuming the emerging hypothetical wavefronts to be circular in a certain vicinity of the surface location under consideration.

Theory. The hypothetical experiments providing the wavefronts of the eigenwaves are illustrated in Figure 1 for a model with three homogeneous layers. We consider a point R on the second interface associated with a normal incidence ray (shown as dashed line) emerging at location x_0 on the surface.

One eigenwave is obtained by placing a point source at R that produces the so-called *normal incidence point* (NIP) wave (Figure 1a, wavefronts depicted in light gray). An exploding reflector experiment yields the second eigenwave called *normal* wave. The wavefronts are again depicted in light gray (Figure 1b). In a vicinity of x_0 both wavefronts are approximated by circles (shown in dark gray) with the radii of curvature R_{NIP} and R_N, respectively.

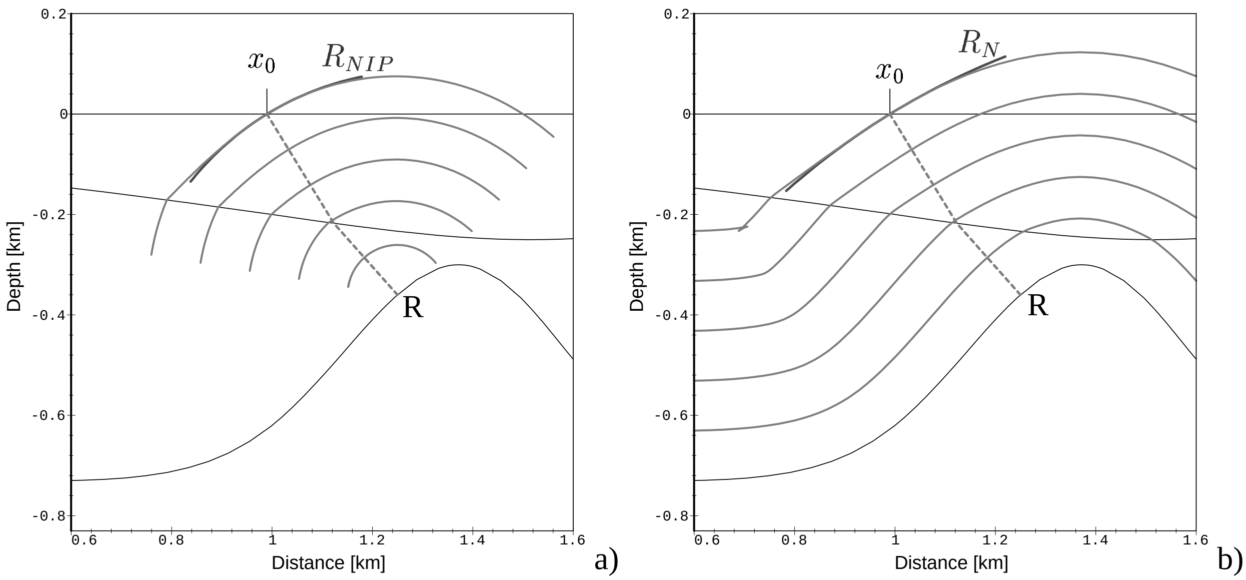

Figure 1: Hypothetical experiments providing a) the NIP wave produced by a point source in R and b) the *normal* wave generated by an exploding reflector experiment. The wavefronts are depicted in light gray, the circular approximations in dark gray. The normal incidence ray (dashed line) is reflected at point R.

An expression of the corresponding stacking operator is available in a parametric form. The parameters are the angle of emergence α of the normal incidence ray, the radius of curvature R_{NIP} of the NIP wave, and the radius of curvature R_N of the *normal* wave.

However, for irregular acquisition geometries an explicit representation of the stacking operator is more convenient. A hyperbolic second order Taylor expansion, which can also be derived by means of paraxial ray theory (Schleicher et al. (1993)), reads

$$t^2\left(x_m\right) = \left(t_0 + \frac{2\,\sin\,\alpha}{v_0}\left(x_m - x_0\right)\right)^2 + \frac{2\,t_0\,\cos^2\,\alpha}{v_0}\left(\frac{\left(x_m - x_0\right)^2}{R_N} + \frac{h^2}{R_{NIP}}\right)\,.$$

The half-offset between source and receiver is denoted with h, whereas x_m denotes the midpoint between source and receiver. The only required model parameter is the near surface velocity v_0. The respective sample of the ZO trace to be simulated is defined by (t_0, x_0).

According to Ursin (1982) and our own experience with different approximations of the CRS stacking operator the hyperbolic approximation with respect to t^2 given above is more appropriate than a parabolic approximation with respect to t. A double square root representation is also possible as shown by Berkovitch et al. (1994).

The proposed strategy can be applied to complex media but is in the presented form based on

ZO rays with normal incidence on the reflector. Furthermore, for inhomogeneous models the CRS stacking operator is only valid in the vicinity of the ZO ray. With regard to ray theory this concerns the paraxial rays of the (central) ZO ray.

Application. For each sample (t_0, x_0) in the stack section, i. e. the zero-offset (ZO) section to be simulated, we have to determine the stacking operator fitting best to an event in the multi-coverage data set: by means of coherence analysis along the stacking operator we look for the parameter triple (α, R_{NIP}, R_N) yielding the highest coherency value. For this example we used the coherence criterion semblance.
We simulated a multi-coverage data set for a model with five dome-like interfaces separating homogeneous layers and added noise with a signal-to-noise ratio of 4. The ZO section of this data set is shown in Figure 2a. The deeper events are hardly visible in this section.
To avoid the time-consuming simultaneous search for all three parameters, we firstly perform a one-parametric search to determine the squared stacking velocity $v_{NMO}^2 = 2\, v_0\, R_{NIP}/(t_0\, \cos^2\alpha)$ in the common midpoint (CMP) gathers. The squared stacking velocity v_{NMO}^2 which may also be negative defines a surface in the (α, R_{NIP}, R_N) domain.
In a second step, a two-parametric search is performed along this surface finally yielding all three parameters. Wherever the coherence exceeds a given threshold an additional three-parametric optimization is applied to improve the accuracy of the parameter triple.
In this way, a stacking operator is defined for each ZO sample to be simulated. The final stack result is shown in Figure 2b where all events can be identified.

Conclusions. The CRS stack is a model independent seismic imaging method and thereby can be performed without any ray tracing and macro velocity model estimation. Only the knowledge of the near surface velocity is required. As a result of a CRS stack one obtains in addition to each simulated ZO reflection time important wave-field attributes: the angle of emergence and the radii of curvature of the *NIP* and the *normal* wave. These attributes can subsequently be used to derive an approximation of the inhomogeneous 2-D macro velocity model (Hubral and Krey (1980), Goldin (1986)) which allows to determine an image in the depth domain.
By means of the Taylor series expansions, the CRS stack can be applied to traces on an arbitrarily irregular grid without the need to interpolate. Additionally, the simulated ZO section and the attribute sections are not restricted to the (possibly irregular) input data geometry.
The application to a synthetic dataset showed noteworthy results with respect to the stack section and the determined attributes. In view of the authors, the proposed strategies offer an exciting approach to improve the stack section and to allow for a subsequent inversion.

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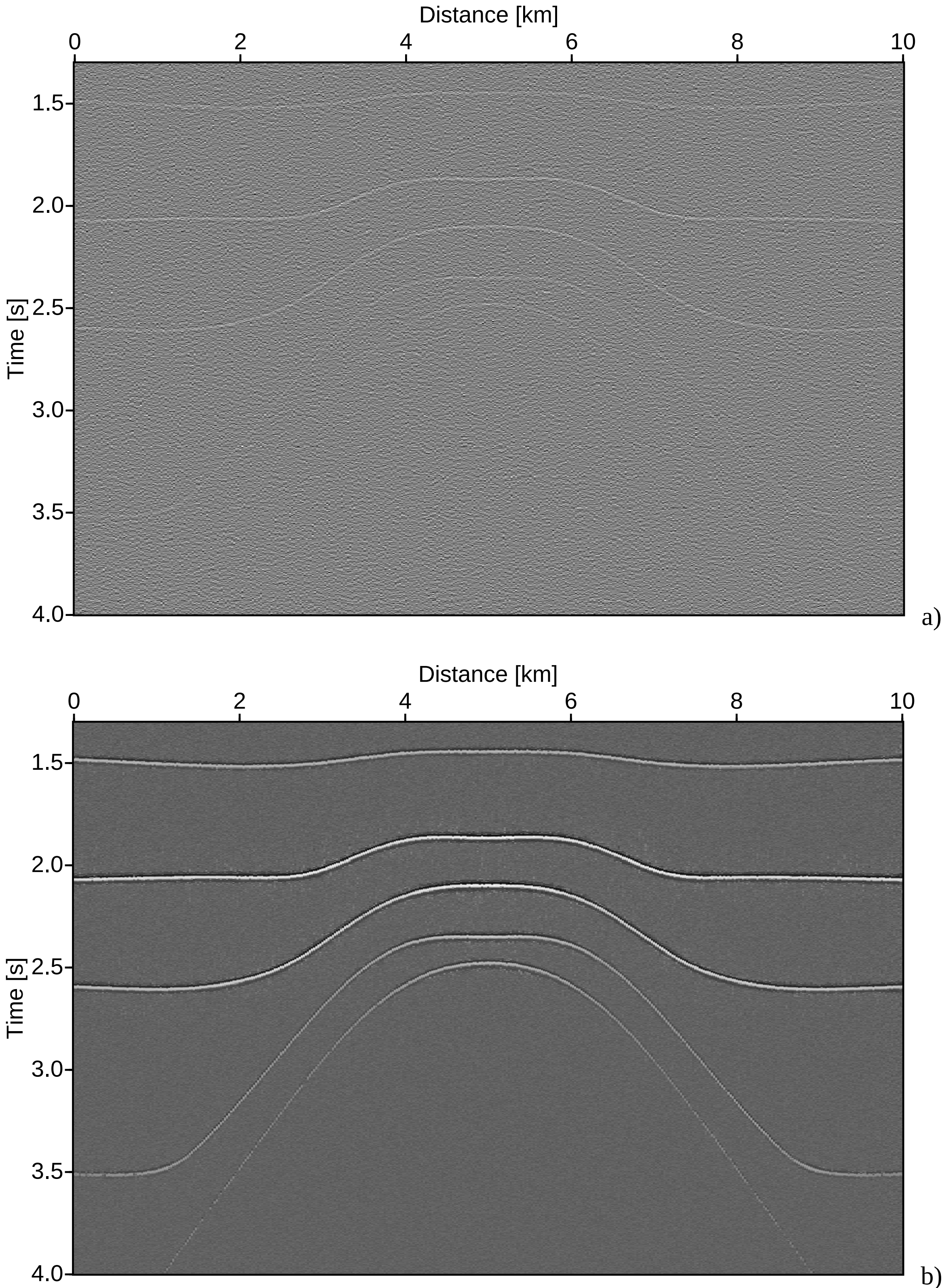

Figure 2: Synthetic data set for a model with five interfaces: a) ZO section of input data set with S/N ratio 4, b) ZO section simulated by means of CRS stack.